SM3 10.3 Series Notation

Expand the series and find the sum:

1) $\sum_{n=1}^{7} 3n - 4$

$-1 + 2 + 5 + 8 + 11 + 14 + 17$

56

2) $\sum_{n=0}^{4} -5n$

$0 - 5 - 10 - 15 - 20$

−50

3) $\sum_{n=5}^{11} (n-1)^2$

$16 + 25 + 36 + 49 + 64 + 81 + 100$

371

4) $\sum_{q=0}^{4} 4(3)^q$

$4 + 12 + 36 + 108 + 324$

484

5) $\sum_{p=1}^{5} \frac{3}{10^p}$

$.3 + .03 + .003 + .0003 + .00003$

$.33333$

6) $\sum_{k=1}^{7} k^3$

$1 + 8 + 27 + 64 + 125 + 216 + 343$

784

7) $\sum_{i=0}^{7} 2(5)^i$

$2 + 10 + 50 + 250 + 1250 +$

$6250 + 31250 + 156250$

195312

8) $\sum_{j=1}^{8} \frac{7}{2^j}$

$\frac{7}{2} + \frac{7}{4} + \frac{7}{8} + \frac{7}{16} + \frac{7}{32} + \frac{7}{64} +$

$\frac{7}{128} + \frac{7}{256}$

$\frac{1785}{256}$

9) $\sum_{m=1}^{6} \sqrt{m}$

$1 + \sqrt{2} + \sqrt{3} + 2 + \sqrt{5} + \sqrt{6}$

$3 + \sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{6}$

Write the series using sigma (Σ) notation:

10) $1 + 4 + 9 + 16 + \ldots + 100$

$$\sum_{n=1}^{10} n^2$$

11) $10 + 50 + 250 + 1250$

$$\sum_{n=1}^{4} 10(5)^{n-1} = \sum_{n=1}^{4} 2(5)^n$$

12) $-1 + 2 + -4 + 8$

$$\sum_{n=1}^{4} -1(-2)^{n-1}$$

13) $1 + -4 + 16 + \ldots + 4096$

$$\sum_{n=1}^{7} (-4)^{n-1}$$

14) $20 + 10 + 5 + \ldots + \frac{5}{8}$

$$\sum_{n=1}^{6} 20\left(\frac{1}{2}\right)^{n-1}$$

15) $-2 + -6 + -18 + \ldots + -162$

$$\sum_{n=1}^{5} -2(3)^{n-1}$$

16) $-1 + \frac{2}{3} + -\frac{4}{9} + \ldots + \frac{32}{243}$

$$\sum_{n=1}^{6} -1\left(-\frac{2}{3}\right)^{n-1}$$

17) $.16 + .8 + \ldots + 100$

$$\sum_{n=1}^{5} .16(5)^{n-1}$$

18) $-128 + 64 + -32$

$$\sum_{n=1}^{3} -128\left(-\frac{1}{2}\right)^{n-1}$$

19) $\frac{2}{5} + \frac{1}{10} + \frac{1}{40} + \ldots \frac{1}{2560}$

$$\sum_{n=1}^{6} \frac{2}{5}\left(\frac{1}{4}\right)^{n-1}$$

20) $.6 + -3 + 15 + -75$

$$\sum_{n=1}^{4} .6(-5)^{n-1}$$

21) $\frac{3}{5} + \frac{9}{5} + \ldots \frac{729}{5}$

$$\sum_{n=1}^{6} \frac{3}{5}(3)^{n-1}$$

22) $x^5 + x^4 + x^3 + x^2 + x + 1$

$$\sum_{n=1}^{6} x^5\left(\frac{1}{x}\right)^{n-1} = \sum_{n=0}^{5} x^{5-n}$$

23) $\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32}$

$$\sum_{n=1}^{5} \frac{3}{2}\left(\frac{1}{2}\right)^{n-1} = \sum_{n=1}^{5} \frac{3}{2^n}$$

Find the sum of the geometric series:

24) $2 + 4 + 8 + 16 + 32 + 64 + 128$

$a_1 = 2; r = 2; n = 7$

$$S_7 = \frac{2(1-2^7)}{1-2} = \frac{2(1-128)}{1-2} = \frac{2(-127)}{-1} = 2(127) = 254$$

25) $\frac{1}{9}+\frac{1}{3}+1+3+\cdots+243$

$a_1=\frac{1}{9}; r=3; n=?$ We need to solve for n

$a_n=a_1r^{n-1}$

$243=\frac{1}{9}(3)^{n-1}$

$3^5=3^{-2}(3)^{n-1}$

$3^5=3^{\mathrm{n}-3}$

$5=\mathrm{n}-3$

$8=\mathrm{n}$ Ok, we've got n. Now let's get the sum

$$S_8=\frac{\frac{1}{9}(1-3^8)}{1-3}=\frac{\frac{1}{9}(1-6561)}{1-3}=\frac{\frac{1}{9}(-6560)}{-2}=\frac{1}{9}(3280)=\frac{3280}{9}\approx 364.4444$$

26) $\frac{1}{2}-\frac{1}{3}+\frac{2}{9}-\frac{4}{27}+\frac{8}{81}$

$a_1=\frac{1}{2}; r=-\frac{2}{3}; n=5$

$$S_7=\frac{\frac{1}{2}\left(1-\left(-\frac{2}{3}\right)^5\right)}{1-\left(-\frac{2}{3}\right)}=\frac{\frac{1}{2}\left(1-\left(-\frac{32}{243}\right)\right)}{1-\left(-\frac{2}{3}\right)}=\frac{\frac{1}{2}\left(1+\frac{32}{243}\right)}{1+\frac{2}{3}}=\frac{\frac{1}{2}\left(\frac{243}{243}+\frac{32}{243}\right)}{\frac{3}{3}+\frac{2}{3}}=\frac{\frac{1}{2}\left(\frac{275}{243}\right)}{\frac{5}{3}}=\frac{1}{2}\left(\frac{275}{243}\right)\left(\frac{3}{5}\right)$$

$$=\frac{1}{2}\left(\frac{55}{81}\right)\left(\frac{1}{1}\right)=\frac{55}{162}\approx 0.3395$$

27) $2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{1024}$

$a_1=2; r=\frac{1}{2}; n=?$ We need to solve for n

$a_n=a_1r^{n-1}$

$\frac{1}{1024}=2\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{2^{10}}=\left(\frac{1}{2}\right)^{n-2}$

$\left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{n-2}$

$10 = n - 2$

$12 = n$ Ok, we've got n. Now let's get the sum

$$S_{12} = \frac{2\left(1-\left(\frac{1}{2}\right)^{12}\right)}{1-\frac{1}{2}} = \frac{2\left(1-\frac{1}{4096}\right)}{1-\frac{1}{2}} = \frac{2\left(\frac{4096}{4096}-\frac{1}{4096}\right)}{\frac{1}{2}} = \frac{2\left(\frac{4095}{4096}\right)}{\frac{1}{2}} = 2\left(\frac{4095}{4096}\right)\left(\frac{2}{1}\right) = \frac{4095}{1024}$$

≈ 3.999

28) $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots + \frac{1}{177147}$

$a_1 = 3; r = -\frac{1}{3}; n = ?$ We need to solve for n

$a_n = a_1 r^{n-1}$

$\frac{1}{177147} = 3\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{3^{11}} = \left(\frac{1}{3}\right)^{n-2}$

$\left(\frac{1}{3}\right)^{11} = \left(\frac{1}{3}\right)^{n-2}$

$11 = n - 2$

$13 = n$ Ok, we've got n. Now let's get the sum

$$S_{13} = \frac{3\left(1-\left(-\frac{1}{3}\right)^{13}\right)}{1-\left(-\frac{1}{3}\right)} = \frac{3\left(1+\frac{1}{1594323}\right)}{1+\frac{1}{3}} = \frac{3\left(\frac{1594324}{1594323}\right)}{\frac{4}{3}} = 3\left(\frac{1594324}{1594323}\right)\left(\frac{3}{4}\right) = \frac{398581}{177147}$$

≈ 2.250001

29) $\sum_{n=1}^{7} 3(2)^{n-1}$

$a_1 = 3; r = 2; n = 7$

$$S_7 = \frac{3(1-2^7)}{1-2} = \frac{3(1-128)}{1-2} = \frac{3(-127)}{-1} = 3(127) = 381$$

30) $\sum_{n=1}^{11} 4(\frac{1}{3})^{n-1}$

$a_1 = 4; r = \frac{1}{3}; n = 11$

$$S_{11} = \frac{4\left(1 - \frac{1}{3}^{11}\right)}{1 - \frac{1}{3}} = \frac{4\left(1 - \frac{1}{177147}\right)}{1 - \frac{1}{3}} = \frac{4\left(\frac{177146}{177147}\right)}{\frac{2}{3}} = 4\left(\frac{177146}{177147}\right)\left(\frac{3}{2}\right) = 4\left(\frac{88573}{59049}\right)\left(\frac{1}{1}\right) = \frac{354292}{59049}$$

≈ 5.999966

31) $\sum_{n=1}^{6} -10\left(\frac{1}{5}\right)^{n-1}$

$a_1 = -10; r = \frac{1}{5}; n = 6$

$$S_6 = \frac{-10\left(1 - \left(\frac{1}{5}\right)^6\right)}{1 - \frac{1}{5}} = \frac{-10\left(1 - \frac{1}{15625}\right)}{1 - \frac{1}{5}} = \frac{-10\left(\frac{15624}{15625}\right)}{\frac{4}{5}} = -10\left(\frac{15624}{15625}\right)\left(\frac{5}{4}\right) = -10\left(\frac{3906}{3125}\right)\left(\frac{1}{1}\right)$$

$= -10\left(\frac{3906}{3125}\right) = -\frac{7812}{625} \approx -12.4992$

32) $\sum_{n=1}^{8} 3(-\frac{1}{4})^{n-1}$

$a_1 = 3; r = -\frac{1}{4}; n = 8$

$$S_8 = \frac{3\left(1 - \left(-\frac{1}{4}\right)^8\right)}{1 - \left(-\frac{1}{4}\right)} = \frac{3\left(1 - \frac{1}{65536}\right)}{1 + \frac{1}{4}} = \frac{3\left(\frac{65535}{65536}\right)}{\frac{5}{4}} = 3\left(\frac{65535}{65536}\right)\left(\frac{4}{5}\right) = 3\left(\frac{13107}{16384}\right)\left(\frac{1}{1}\right)$$

$= \frac{39321}{16384} \approx 2.3999$

33) Your fridge is running low on milk. The last milk carton has only 1 cup of milk remaining. Your family has a rule that whoever drinks the last of the milk has to go buy more milk, and you don't want that to be you. So each morning, you just drink half of the remaining milk! After a week, your parent notices the game you are playing and then makes you go buy more milk with your own money for being such a pain and breaking the spirit of the house rule. Write the sequence of how much milk you drank each day as sequence m. Write the series s that sums the values of sequence m using sigma notation. Use the series formula to determine how much milk did you drink during the week where you attempted to avoid having to buy more milk?

$$m = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}$$

$$s = \sum_{k=1}^{7} \frac{1}{2^k} = \frac{\frac{1}{2}\left(1 - \left(\frac{1}{2}\right)^7\right)}{1 - \frac{1}{2}} = \frac{\frac{1}{2}\left(1 - \frac{1}{128}\right)}{\frac{1}{2}} = \frac{127}{128}$$